

Optimal Balancing of Spatial Suspended Cable Robot in Point-to-Point Motion using Indirect Approach

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Abstract: In this paper, a method based on the indirect solution of optimal control problem is presented to specify the optimal trajectory of spatially suspended cable robot in point to point motion with considering the counterweights. In fact, an optimal trajectory planning problem is outlined in which states, controls and the values of counterweights must be calculated simultaneously in order to minimize the given performance index. The value of the pulley torques is considered for the performance index (objective function). Using the fundamental theorem of a calculus of variations, the necessary conditions for optimality of cable robot are achieved. For the three-cable spatial robot, a two-point boundary value problem is achieved which can be solved with `bvp4c` command in MATLAB. The obtained results show that optimal balancing in comparison with the unbalancing method can reduce the performance index significantly.

Keywords: Cable robot, Counterweights, Optimal balancing, Point-to-Point motion

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1 INTRODUCTION

Cable robots are a type of parallel manipulator wherein the end-effector is supported by n cables with n tensioning motors. In addition to the well-known advantages of parallel robots relative to serial robots, cable robots also have very low mass and even better stiffness than other parallel robots. In recent years, path planning of cable robots is well studied. Two strategies can be defined for path optimization: direct and indirect methods. These techniques are used in enormous articles and they have own benefits and drawbacks [1]. Besides the trajectory planning in which controls and states are obtained to minimize the desired performance index, one efficient way to increase the robot performance is balancing [2].

Trajectory planning problem can be stated as an optimal control problem usually solved by direct methods. Direct method converts the trajectory planning problem into a parameter optimization problem accomplished by discretization of dynamic variables including states and controls [3]. In some of the previous works dealing with path planning of robotic manipulators the direct methods are employed, and often the Spline or polynomial functions are used as the motion profiles [4], [5]. Lahouar et al., used direct method for collision-free path planning of cable-driven parallel robots [6]. On the other hand, indirect methods are based on Pontryagin Minimum Principle (PMP), which solves the optimal control problem exactly [7].

Based on PMP, the problem of optimal control is first transformed into multipoint boundary value problem. In the next step, this problem is discretized to attain the numerical solution using the method such as shooting, relaxation or collocation techniques. The indirect method was first applied to solve the minimum time motion problems along the specified paths. Then, it was extended to handle free motions as well [8]. PMP is also used to solve directly the optimal dynamic motion planning problem [9]. Furuno et al. used this technique for trajectory planning of mobile manipulators [10]. Korayem et al., used this method for trajectory planning of flexible joint [11], flexible link [12] and redundant manipulators [13]. Korayem et al. designed a computational approach for achieving optimal trajectory to maximize dynamic load capacity of cable robot in point-to-point motion [14].

Korayem et al., used the open loop optimal control method for generating the optimal trajectory [15]. Balancing introduces some modifications in the architecture of the original mechanism, which actually simplifies its dynamic model and, as a result, its control as well. Besides control simplification, balancing can also provide a reduction of driving torques. Balancing can be classified into either static or dynamic. Static

balancing means that the weight of the members does not produce any force at actuators for any configuration of the manipulator [16]. In dynamic balancing, some modifications are applied to the original kinematic chain of unbalanced mechanisms to achieve static balancing and complete decoupling of dynamic equations [17]. Thus coriolis, centripetal, gravitational and cross inertia terms are eliminated. Furthermore, mechanical balancing of mechanisms has received sustained interest from researchers, since it allows one to significantly decrease the size of actuators for equivalent displacements of the end effector.

This role is generally rather fulfilled by springs, counterweights, pneumatic or hydraulic cylinders, and even by electromagnetic devices. Springs do not affect much the mechanism's inertia. Thus, they are widely used in balancing. As example, Nikoobin et al., used the optimal spring balancing for robot manipulators [18] or Perreault et al. which has developed a nonlinear spring to maintain a given minimum tension in the cables of a parallel cable-driven mechanism [19]. Even if, in general, counterweights add inertia to the mechanism, they have been extensively used to balance the mechanisms. Nikoobin et al., presented a balancing approach for open chain robot manipulators using the open loop optimal control by counterweights [20] or in the other work, the indirect method is used to determine the optimal trajectory of planar cable robot in point to point motion [21]. The results of literature survey are summarized as follows:

A weak point of the direct methods is that it leads to an approximately optimal solution. Furthermore, they are exhaustively time-consuming due to the large number of parameters involved, especially for systems with a large number of degree of freedoms [22]. The indirect methods are widely used as a powerful and efficient tool for analyzing the nonlinear systems and path planning of different types of systems [23], [24]. In the balancing method, the value of the spring constants or the counterweights are achieved without considering the trajectory [16], [17], [19].

On the other side, in the optimal balancing method, the value of the spring constants or the counterweights is obtained with considering the trajectory. The optimal balancing method has been applied to the serial robot and planar cable robot [18], [20], [21], but this method has not been applied on spatial cable robot yet. In this paper, a method based on the indirect solution of optimal control problem is introduced to specify the optimal trajectory of spatially suspended cable robot in point to point motion with considering the counterweights. In the proposed method called optimal balancing, states, controls and the values of counterweights are obtained simultaneously, in which the objective function is minimized.

2 DYNAMIC MODELING OF ROBOT

In this section, the dynamic equation of robot is extracted [25], [26]. A cable-suspended robot typically consists of a moving platform that is connected to a fixed base by several cables [27]. The spatial suspended cable robot including the counterweights is shown in Fig. 1. As it can be seen, three compensation masses mc_1, mc_2 and mc_3 are attached to cables 1, 2 and 3, respectively. The cross section of the robot workspace is an equilateral triangle with side length a . The global coordinate system is placed in the confluence of the medians of this triangle. The end effector of the robot with mass m has three degrees of freedom as $X=[x \ y \ z]^T$. Moreover, the acceleration of gravity is in the direction z and the downward. The system consists of three motors, three pulleys and three cables. The coordinate of pulleys is as follow:

$$P_1 = [-\frac{a}{2} \ -\frac{b}{3} \ h]^T, P_2 = [\frac{a}{2} \ -\frac{b}{3} \ h]^T, P_3 = [0 \ \frac{2b}{3} \ h]^T; b = \sqrt{a^2 - (\frac{a}{2})^2}. \tag{1}$$

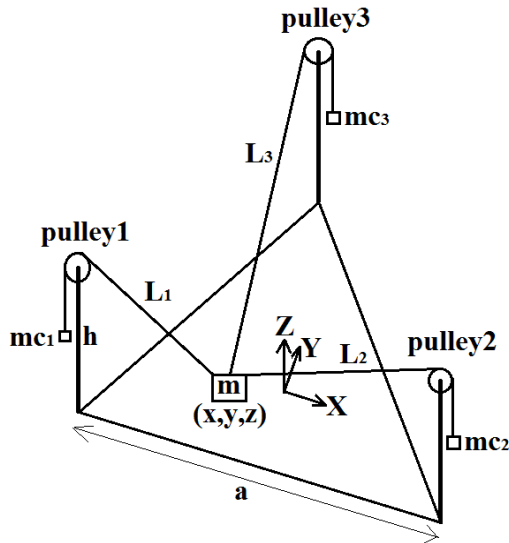


Fig. 1 Three cable robot with counterweights.

At first, the cables length is determined by following equation

$$L_i = \sqrt{(x - P_{ix})^2 + (y - P_{iy})^2 + (z - P_{iz})^2}; i = 1, 2, 3. \tag{2}$$

By using the cables length, the Jacobian matrix can be calculated as follow

$$J = \frac{\partial L_i}{\partial X} = \begin{bmatrix} \frac{\partial L_1}{\partial x} & \frac{\partial L_1}{\partial y} & \frac{\partial L_1}{\partial z} \\ \frac{\partial L_2}{\partial x} & \frac{\partial L_2}{\partial y} & \frac{\partial L_2}{\partial z} \\ \frac{\partial L_3}{\partial x} & \frac{\partial L_3}{\partial y} & \frac{\partial L_3}{\partial z} \end{bmatrix}. \tag{3}$$

Also the linear velocity of cables is calculated as follow:

$$\dot{L}_i = J_{(i)} \dot{X}; \dot{X} = [\dot{x} \ \dot{y} \ \dot{z}]^T, \tag{4}$$

where \dot{X} is the linear velocity of the end effector in direction x, y and z . Rotating the i^{th} pulley leads to change of the i^{th} cable length. Therefore, the pulleys angles are obtained as follow

$$\beta_i = \frac{1}{r}(L_{0i} - L_i), L_{0i} = \sqrt{P_{ix}^2 + P_{iy}^2 + P_{iz}^2}; i = 1, 2, 3, \tag{5}$$

where L_{0i} is the initial length of the cables. By derivation of the pulley angle with respect to time, angular velocity of pulleys is obtained as follow

$$\dot{\beta}_i = \frac{\partial \beta_i}{\partial X} \frac{dX}{dt} = \frac{-1}{r} \frac{\partial L_i}{\partial X} \frac{dX}{dt} = \frac{-1}{r} J \dot{X}; i = 1, 2, 3. \tag{6}$$

Now, the positions of compensation masses mc_1, mc_2 and mc_3 are described. The initial position of the compensation masses is $2b/3$ according to the location of the end effector in coordinate $X=[0 \ 0 \ 0]^T$ (Fig. 2). Therefore, the position of three compensation masses is obtained in terms of the global coordinate system by following equation

$$z_i = h - \frac{2b}{3} + \Delta L_i, \Delta L_i = L_i - L_{0i}; i = 1, 2, 3. \tag{7}$$

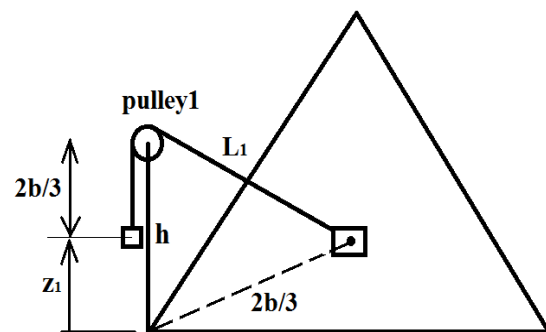


Fig. 2 Position of mc_1 according to the location of the end-effector in $X=[0 \ 0 \ 0]^T$.

The Lagrange's method is used to obtain the dynamic equation. This method is written in terms of the potential energy U , the kinetic energy K and the generalized force or torque Q , as follow:

$$\frac{d}{dt} \left(\frac{\partial K(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial K(q, \dot{q})}{\partial q} + \frac{\partial U(q)}{\partial q} = Q. \quad (8)$$

Parameter q in Eq. (8) is called generalized coordinate. By the following steps, the dynamic equation of the end effector is obtained.

Step 1- The equation of motion for mass m :

The kinetic energy, the potential energy, and the generalized force are obtained as follow

$$K = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2), U = mgz \quad (9)$$

$$Q = -J^T [T_1 \ T_2 \ T_3]^T = -J^T T$$

where g , J and T are gravity acceleration, Jacobian matrix and cable tension vector, respectively. By using Eq. (8), the dynamic equation is obtained as follow

$$M\ddot{X} + G = -J^T T \quad (10)$$

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}, \ddot{X} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}, G = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}.$$

Step 2- The equation of motion for counterweights mc_1 , mc_2 and mc_3 :

The dynamic equation is obtained as follow

$$M_c \ddot{Z}_c + G_c = T$$

$$M_c = \begin{bmatrix} mc_1 & 0 & 0 \\ 0 & mc_2 & 0 \\ 0 & 0 & mc_3 \end{bmatrix}, \ddot{Z}_c = \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \end{bmatrix}, G_c = \begin{bmatrix} mc_1 g \\ mc_2 g \\ mc_3 g \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}. \quad (11)$$

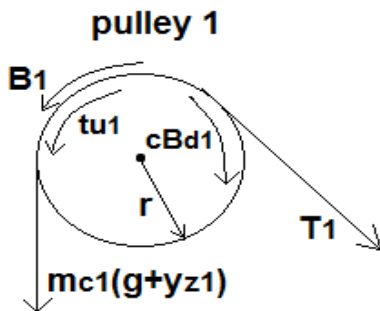


Fig. 3 Pulley 1.

Step 3- The equation of motion for Pulleys 1, 2 and 3: The kinetic energy and the generalized torque of pulley 1 is obtained by Eq. (12), according to Fig. 3,

$$K = \frac{1}{2} j' \dot{\beta}_1^2, Q = \tau_1 - rT_1 - c\dot{\beta}_1 + rmc_1(g + \ddot{z}_1) \quad (12)$$

where β_1 and j' are pulley angle 1 and moment of inertia of pulley, respectively. Also, τ_1 , r and c are motor torque, pulley radius and viscous damping coefficient, respectively. By using Eq. (8), the dynamic equation is obtained as follow

$$j' \ddot{\beta}_1 = \tau_1 - rT_1 - c\dot{\beta}_1 + rmc_1(g + \ddot{z}_1). \quad (13)$$

Thus, cable tension 1 is derived as

$$T_1 = \frac{1}{r} (\tau_1 - j' \ddot{\beta}_1 - c\dot{\beta}_1 + rmc_1(g + \ddot{z}_1)). \quad (14)$$

Similarly, the cable tensions 2 and 3 is derived. Therefore, cable tension is written as follow

$$T = \frac{1}{r} (\tau - j\ddot{\beta} - C\dot{\beta} + rM_c \ddot{Z}_c + rG_c) \quad (15)$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}, j = \begin{bmatrix} j' & 0 & 0 \\ 0 & j' & 0 \\ 0 & 0 & j' \end{bmatrix}, \ddot{\beta} = \begin{bmatrix} \ddot{\beta}_1 \\ \ddot{\beta}_2 \\ \ddot{\beta}_3 \end{bmatrix},$$

$$C = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}, \dot{\beta} = \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{bmatrix}.$$

Now, by multiplying the Eq. (15) by $-J^T$, following equation is obtained.

$$-J^T \times \text{Eq. (15)} \rightarrow$$

$$-J^T T = \frac{-J^T}{r} (\tau - j\ddot{\beta} - C\dot{\beta} + rM_c \ddot{Z}_c + rG_c) \quad (16)$$

Then, by substituting the Eq. (10) into Eq. (16), one can write

$$rM\ddot{X} + rG - J^T j\ddot{\beta} - J^T C\dot{\beta} + rJ^T (M_c \ddot{Z}_c + G_c) = -J^T \tau$$

$$\ddot{\beta} = \frac{d}{dt} \left(\frac{\partial \beta}{\partial X} \right) \dot{X} + \frac{\partial \beta}{\partial X} \ddot{X}, \dot{Z}_c = \dot{L} = J\dot{X} \rightarrow \ddot{Z}_c = J\ddot{X} + J\dot{X} \quad (17)$$

Finally, the final dynamic equation is achieved as follow

$$M_i \ddot{X} + N_i + G_i = -J^T \tau$$

$$M_i = r(M + J^T M_c J) - J^T j \frac{\partial \beta}{\partial X}$$

$$N_i = -J^T \left(j \frac{d}{dt} \left(\frac{\partial \beta}{\partial X} \right) + C \frac{\partial \beta}{\partial X} - rM_c J \right) \dot{X}$$

$$G_i = r(G + J^T G_c), \quad (18)$$

Where M_i , N_i and G_i are the inertia matrix, the Coriolis and centrifugal terms, and gravity term, respectively.

3 FORMULATION OF OPTIMAL BALANCING

In optimal balancing approach presented in this article, the values of counterweights, trajectory of robot and applied torque in each pulley must be obtained simultaneously in such a way that a given performance index is minimized. For this purpose, compensation masses are assumed as unknown variables, and then dynamic equations for robot manipulator are derived. By considering the counterweights vector denoted by mc_i , Eq. (18) is obtained. Now, optimal balancing problem can be solved. So, by defining the continuous state vector as

$$x = [x_1 \ x_2]^T = [q \ \dot{q}]^T = [[x \ y \ z] \ [\dot{x} \ \dot{y} \ \dot{z}]]^T, \quad (19)$$

The dynamic equation (18) can be rewritten in state space form as

$$f = \dot{x} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ M_i^{-1}(x_1, mc) [-J^T \tau - N_i(x, mc) - G_i(x_1, mc)] \end{bmatrix}, \quad (20)$$

Where f is continuous in the variables x , u and t , and is continuously differentiable with respect to x . Then Hamiltonian function can be written as follow

$$H_{mc} = L(x, u, mc) + \psi^T(t) f(x, u, mc), \quad (21)$$

Where $L(x, u, mc)$ is Lagrange function and $\psi = [\psi_1 \ \psi_2]^T$ denotes co-state vector. Substituting for $f(x, u, mc)$ from Eq. (20) into Eq. (21) gives

$$H_{mc} = L(x, u, mc) + \psi_1^T(t) x_2 + \psi_2^T(t) [M_i^{-1}(x_1, mc) [-J^T \tau - N_i(x_1, x_2, mc) - G_i(x_1, mc)]] \quad (22)$$

Using the PMP, one can write the optimality conditions as follow [7]

$$\dot{x} = f(x, u, mc), \dot{\psi} = -H_x^T, H_u = 0, \dot{\mu} = -H_{mc}^T \quad (23)$$

$$x(t_0) = x_0, \mu(0) = 0, x(t_f) = x_f, \mu(t_f) = 0,$$

Now, by substituting the Hamiltonian function (22) into Eq. (23), the optimality conditions become :

$$\dot{x}_1 = x_2, \dot{x}_2 = M_i^{-1}(x_1, mc) [-J^T \tau - N_i(x_1, x_2, mc) - G_i(x_1, mc)], \quad (24)$$

$$\dot{\psi}_1 = -\left[\frac{\partial L}{\partial x_1} + \psi_2^T(t) \times \frac{\partial [M_i^{-1}(x_1, mc) [-J^T \tau - N_i(x_1, x_2, mc) - G_i(x_1, mc)]]}{\partial x_1} \right],$$

$$\dot{\psi}_2 = -\left[\frac{\partial L}{\partial x_2} + \psi_1^T(t) + \psi_2^T(t) \times \frac{\partial [-N_i(x_1, x_2, mc)]}{\partial x_2} \right], \quad (25)$$

$$\dot{\mu} = -\left[\frac{\partial L}{\partial mc} + \psi_2^T(t) \times \frac{\partial [M_i^{-1}(x_1, mc) [-J^T \tau - N_i(x_1, x_2, mc) - G_i(x_1, mc)]]}{\partial mc} \right], \quad (26)$$

$$\frac{\partial L}{\partial \tau} - \psi_2^T(t) M_i^{-1}(x_1, mc) J^T = 0, \quad (27)$$

$$x(t_0) = x_0, \mu(0) = 0, x(t_f) = x_f, \mu(t_f) = 0, \quad (28)$$

Where for a 3-dof robot, Eq. (24) represents 6 equations dealing with states, Eq. (25) represents 6 equations dealing with co-states, Eq. (26) represents 3 equations dealing with auxiliary states μ related to counter weights, Eq. (27) represents 3 equations dealing with controls and Eq. (28) represent 18 (12+6) boundary conditions. By substituting the control value u obtained of Eq. (27) into Eqs. (24), (25) and (26), a set of 15 ordinary differential equations is established which beside the 18 boundary value conditions given in Eq. (28), forms a two-point boundary value problem. Finally, the derived TPBVP is solved to obtain 6 state, 6 costate, 3 counterweight value, and 3 auxiliary state μ .

For the spatial robot, two different conditions are considered: unbalanced and optimally balanced. In unbalanced case, counterweights of robot are zero ($mc_1=mc_2=mc_3=0$). In optimal balanced case the values of counterweights are depended on dynamic equations, performance index and boundary conditions according to Eqs. (24-28).

The initial position of the end-effector at $t=0$ is (x_0, y_0, z_0) and the final position at $t=t_f$ is (x_f, y_f, z_f) . The initial and final velocity is considered to be zero. So, one can write the boundary conditions as follow

$$x(0) = x_0, y(0) = y_0, x(t_f) = x_f, y(t_f) = y_f, z(0) = z_0, z(t_f) = z_f, \dot{x}(0) = \dot{y}(0) = \dot{z}(0) = \dot{x}(t_f) = \dot{y}(t_f) = \dot{z}(t_f) = 0. \quad (29)$$

At the first step, by defining the continuous state vector as follow

$$X_1 = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, X_2 = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix}, \quad (30)$$

The state space from of equations, using Eq. (20), becomes

$$\dot{x}_1 = x_4, \dot{x}_2 = x_5, \dot{x}_3 = x_6, \begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = M_i^{-1}(X_1, mc) [-J^T \tau - N_i(X_1, X_2, mc) - G_i(X_1, mc)]. \quad (31)$$

Now by considering the performance index as minimum control effort which is defined as follows:

$$J = \int_0^{t_f} L(x, u, mc) dt = \int_0^{t_f} (\tau_1^2 + \tau_2^2 + \tau_3^2) dt, \quad (32)$$

And the co-state vector as $\psi_1 = [x_7 \ x_8 \ x_9]^T$ and $\psi_2 = [x_{10} \ x_{11} \ x_{12}]^T$, the Hamiltonian function using Eq. (22) becomes:

$$H = (\tau_1^2 + \tau_2^2 + \tau_3^2) + x_7 \dot{x}_1 + x_8 \dot{x}_2 + x_9 \dot{x}_3 + x_{10} \dot{x}_4 + x_{11} \dot{x}_5 + x_{12} \dot{x}_6, \quad (33)$$

Where \dot{x}_i ; $i=1, \dots, 6$ can be substituted from Eq. (31). Then by substituting Eq. (31) into Eq. (33), and differentiating the Hamiltonian function with respect to the states, according to Eq. (25), the co-state equations are obtained as follows:

$$\dot{\psi} = [\dot{x}_7 \ \dot{x}_8 \ \dot{x}_9 \ \dot{x}_{10} \ \dot{x}_{11} \ \dot{x}_{12}]^T. \quad (34)$$

After that, using Eq. (27), the control values can be obtained by solving the following equations as :

$$\frac{\partial H}{\partial \tau_1} = 0, \frac{\partial H}{\partial \tau_2} = 0, \frac{\partial H}{\partial \tau_3} = 0. \quad (35)$$

By solving Eq. (35), the optimal torques are obtained based on states, co-states, and counterweight parameters. Thus, the optimal torques are derived by the optimum trajectory and the optimal values of counterweights.

Dynamic equations, co-state equations and optimal control law are the same as obtained in Eqs. (31), (34) and (35), respectively. Here, in all equations mc_1 , mc_2 and mc_3 are considered to be unknown parameters.

Now using Eq. (26), by defining the three new state variables x_{13} , x_{14} and x_{15} , the optimality conditions associated with the parameters are given by :

$$\dot{x}_{13} = -\frac{\partial H}{\partial mc_1}, \dot{x}_{14} = -\frac{\partial H}{\partial mc_2}, \dot{x}_{15} = -\frac{\partial H}{\partial mc_3}, \quad (36)$$

Where according to Eq. (28) the associated boundary conditions become

$$x_{13}(0) = x_{14}(0) = x_{15}(0) = x_{13}(t_f) = x_{14}(t_f) = x_{15}(t_f) = 0. \quad (37)$$

At last, by substituting Eq. (35) into Eqs. (31), (34) and (36), 15 nonlinear ordinary differential equations with respect to states $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$, co-states $[x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12}]^T$, new states $[x_{13} \ x_{14} \ x_{15}]^T$, and unknown parameters $[mc_1 \ mc_2 \ mc_3]^T$ will be achieved. These fifteen equations with eighteen boundary conditions given in Eq. (29) and Eq. (37), construct a two-point boundary value problem which can be solved using the bvp4c command in MATLAB.

4 SIMULATION RESULTS

All required parameters of the robot are given in Table 1. For simulation, two paths are considered.

Table 1 The Robot parameters

$a(m)$	$h(m)$	$m(kg)$	$r(m)$	$j(kg.m^2)$	$c(N.m.s)$	$g(m/s^2)$
2	1.5	3	0.03	0.001	0.01	9.81

4.1. Path 1

The path is shown in Table 2.

Table 2 The properties of path for spatial cable robot

Position (m)	Velocity (m/s)	Time (s)
$x_0=0.5, y_0=-0.3,$ $z_0=1.1$	$Vx_0=Vy_0=Vz_0=0$ $Vx_f=Vy_f=Vz_f=0$	$t_f=1$
$x_f=0.1, y_f=0.45,$ $z_f=0.3$		

By solving the two-point boundary value problem obtained in Section 3, the values of objective function obtained from Eq. (32) are shown in Table 3. As it can be seen, performance index for optimal balanced case is smaller than the unbalanced case significantly.

Table 3 The objective function value

	Objective function $((N.m)^2.s)$
Unbalanced	0.6186
Optimal balanced	0.1600

The optimal value of the counterweights is illustrated in Table 4. The position of the end effector is illustrated in Fig. 4 and the velocity of the end effector is shown in Fig. 5.

Table 4 The optimal value of the compensation masses

Counterweights	mc_1	mc_2	mc_3
	0.9420	0.0908	2.0961

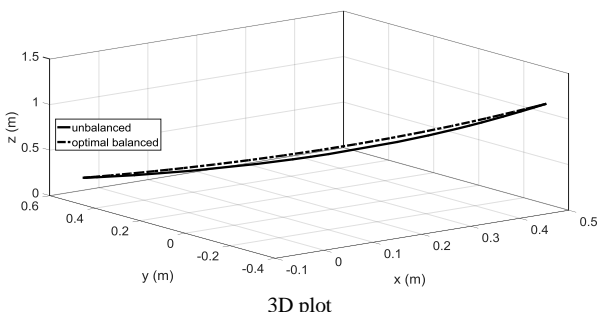
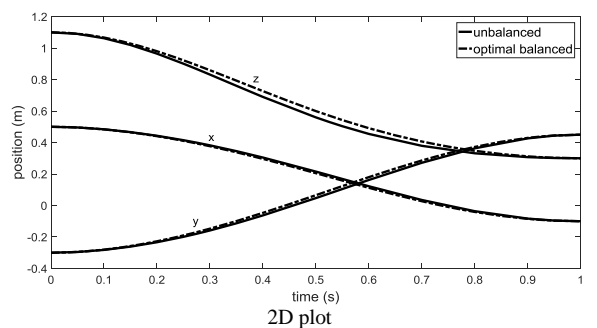


Fig. 4 The position of end effector.

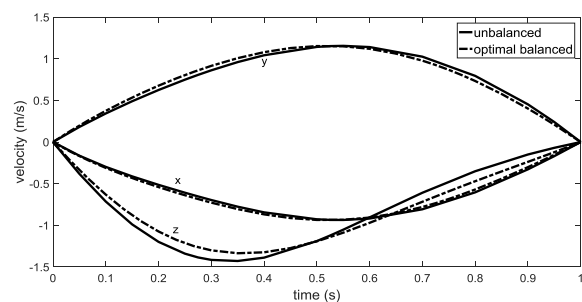


Fig. 5 The velocity of end effector.

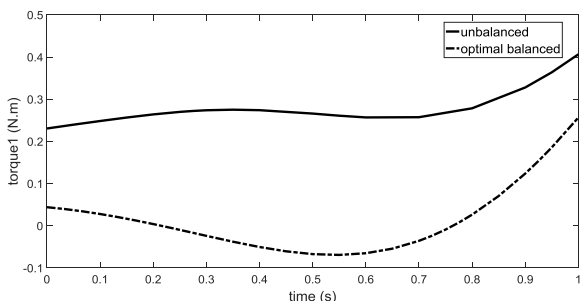


Fig. 6 The torque of pulley 1.

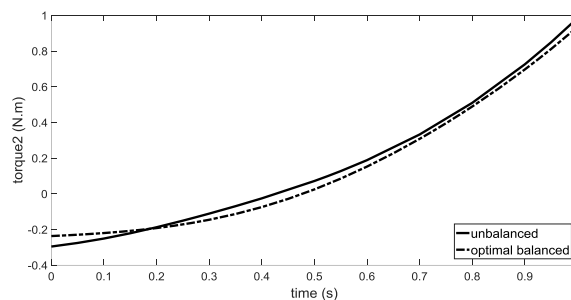


Fig. 7 The torque of pulley 2.

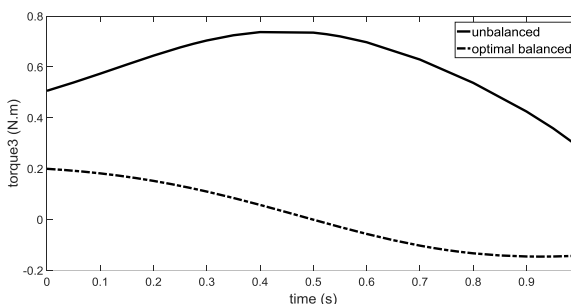


Fig. 8 The torque of pulley 3.

The torque for unbalanced and optimal balanced cases is plotted in Figs. 6-8. As it can be seen the applied torque for optimal balanced case is lower than the unbalanced case. The cable tensions are obtained by Eq. (15). The three cable tensions are shown in Fig. 9.

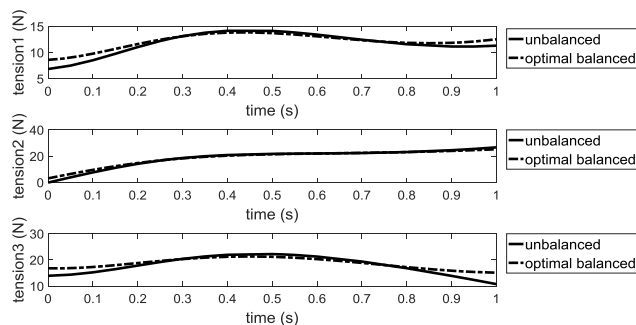


Fig. 9 The tension of cables.

4.2. Path 2

The path is shown in Table 5.

Table 5 The properties of path for spatial cable robot

Position (m)	Velocity (m/s)	Time (s)
$x_0 = -0.4, y_0 = -0.2,$ $z_0 = 0.55$	$V_{x_0} = V_{y_0} = V_{z_0} = 0$	$t_f = 1$
$x_f = 0.35, y_f = 0.05,$ $z_f = 1.2$	$V_{x_f} = V_{y_f} = V_{z_f} = 0$	

By solving the two-point boundary value problem obtained in Section 3, the values of objective function obtained from Eq. (32) are shown in Table 6. As it can be seen, performance index for optimal balanced case is smaller than the unbalanced case significantly.

Table 6 The objective function value

	Objective function $((N.m)^2.s)$
Unbalanced	1.5445
Optimal balanced	0.1691

The optimal value of the counterweights is illustrated in Table 7. The position of the end effector is illustrated in Fig. 10 and the velocity of the end effector is shown in Fig. 11.

Table 7 The optimal value of the compensation masses

Counterweights	mc_1	mc_2	mc_3
	1.7632	2.8214	2.4851

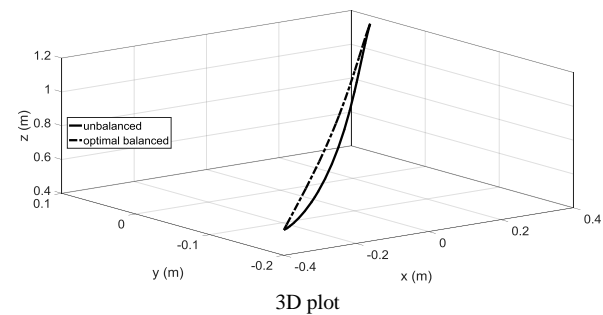
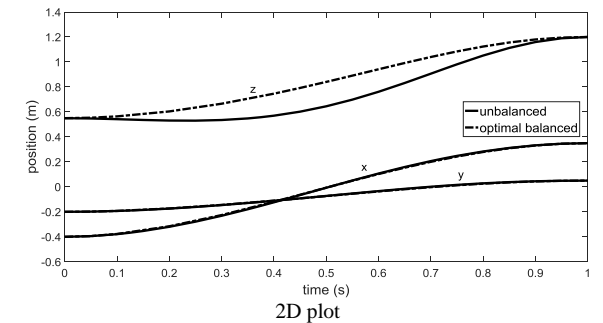


Fig. 10 The position of end effector.

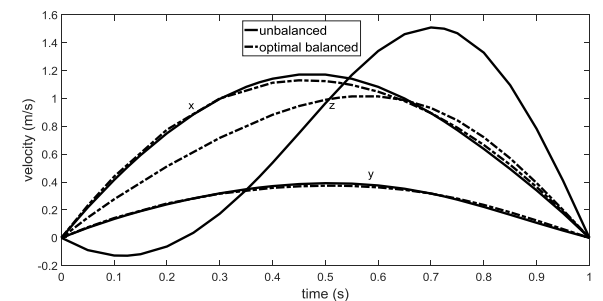


Fig. 11 The velocity of end effector.

The torque for unbalanced and optimal balanced cases is plotted in Figs. 12-14. As it can be seen, the applied torque for optimal balanced case is lower than the unbalanced case.

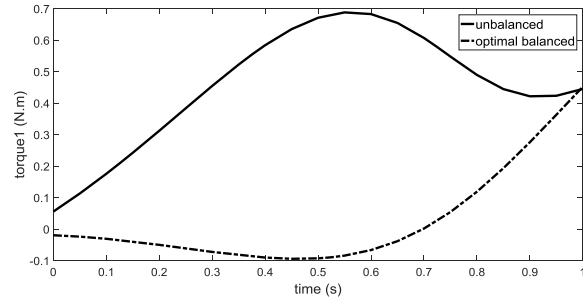


Fig. 12 The torque of pulley 1.

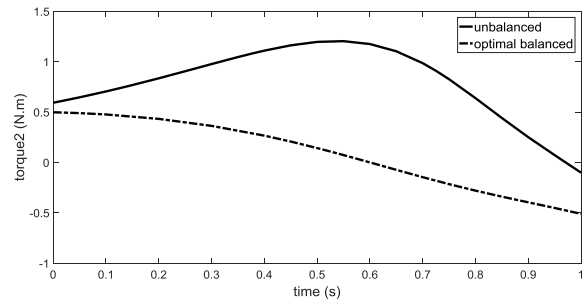


Fig. 13 The torque of pulley 2.

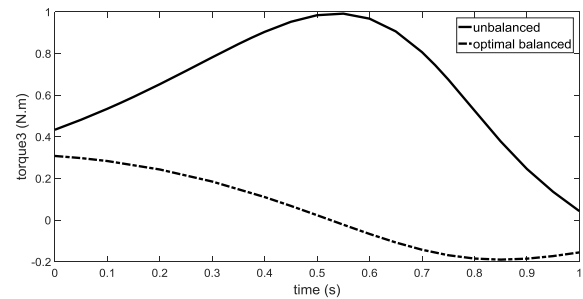


Fig. 14 The torque of pulley 3.

The cable tensions are obtained by Eq. (15). The three cable tensions are shown in Fig. 15.

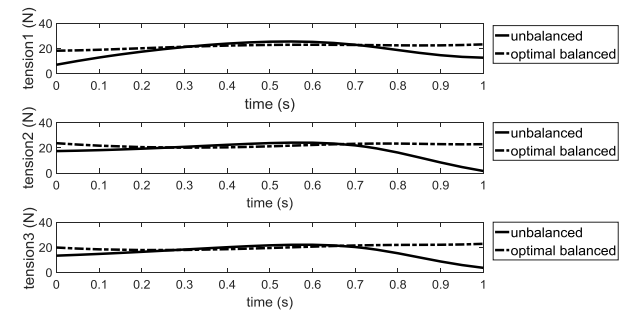


Fig. 15 The tension of cables.

5 CONCLUSION

One important contribution of the optimal balancing is that states, controls, and the values of counterweights are determined simultaneously in order to minimize the given performance objective, by solving the equations obtained of optimality conditions. In this article, a new area for balancing and trajectory planning is suggested which results in the best possible response. The efficiency of the proposed method is investigated through computer simulations by considering a spatial three-cable robot. The obtained results show that the performance index can be reduced significantly if the values of counterweights are chosen properly.

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