

# Design of a Dynamically Balanced 2-DOF Planar Parallel Manipulator using Four-bar Legs

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**Abstract:** A mechanism is reactionless or dynamically balanced when there is no shaking force and shaking moment applied to the base during mechanism movement. The theory for designing reactionless 2 degree-of-freedom (DOF) planar parallel manipulator is discussed in this paper. The legs of the manipulator are four-bar 2-DOF mechanisms with revolute joints. The dynamic balancing conditions of the manipulator are derived, considering that the time rate of the total linear and angular momentum have to be vanished. The dynamic balancing equations first are obtained and illustrated through a numerical example and finally verified by computer simulation using ADAMS software.

**Keywords:** 2 Degree of freedom mechanism, ADAMS, Dynamic balance, Manipulator

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## 1 INTRODUCTION

Manipulators could be classified according to the type of kinematic chains which are used for their implementations. Serial manipulators are based on open kinematic chains, while parallel manipulators are based on closed kinematic chains. Each type of robotic manipulator has its advantages and drawbacks and is suitable for different applications. In order to their abilities, parallel manipulators have received considerable attentions by researches [1-4]. As compared with serial manipulators, parallel manipulators are characterized by low moving inertia, high stiffness, high dexterity, compact size and high power to weight ratio and hence they can be controlled with a high bandwidth. Parallel manipulators are excellent for applications requiring large load capacity or high speed and accuracy [5]. However, similarly to other robotic devices, they exert forces and moments on their base during movement, causing vibration and associated noise, wear, fatigue as well as disturbances in the supporting structure of the mechanism [6]. These reaction forces and moments, so-called shaking forces and shaking moments, are not desired in many applications. In order to eliminate the undesired shaking forces and moments, reactionless mechanism has been proposed. A mechanism is said to be reactionless or dynamically balanced if for any motion of the mechanism, there is no shaking force and moments at its base at all-time [5].

Static balancing is an approach that has to be considered former to dynamic balancing. A mechanism is statically balanced while potential energy is constant for all possible configurations. This property is very convenient for robotic manipulators with large constant payloads, since it means that the mechanism is statically stable for any configuration, i.e. zero actuator torques are required whenever the manipulator is at rest. Interested readers may be referred to [7-9], [4]. In order to obtain dynamically balanced manipulators two major conditions must be satisfied:

- 1) Time rate of linear momentum of the manipulator should be vanished, so the center of mass of the system must be inertial fixed.
- 2) Time rate of angular momentum of the manipulator about the center of mass of the system has to be vanished, too. The former is equal to resultant shaking forces, while the later is resultant shaking moments.

Regarding the importance of dynamic behavior of parallel manipulators and based on the literature, there are many researches on the dynamic balancing of the parallel manipulators. As in [10], authors proposed a method for dynamic balancing of Hexapods for high-

speed application. The proposed method was aimed at minimizing the changes in the Hexapods inertia over the workspace. Optimum dynamic balancing of planar parallel manipulators has been addressed by [11], based on sensitivity analysis. They addressed the optimum dynamic balancing of planar parallel manipulators exemplified with 2-DOF parallel manipulator articulated with revolute joints. The dynamic balancing was formulated as an optimization problem such that while shaking force balancing is accomplished through analytically obtained balanced constraints, an objective function based on the sensitivity analysis of shaking moment with respect to the position, velocity and acceleration of the links is used to minimize the shaking moment.

Reference [12] presented the synthesis of novel reactionless spatial 3-DOF and 2-DOF mechanisms without any separate counter-rotation, using four-bar linkages. Reference [13] also addressed the design of reactionless 3-DOF and 2-DOF parallel manipulators using 3-DOF parallel mechanism (Parallelepiped) as legs. Reference [14] presented the theory of design of a reactionless 3-DOF planar parallel manipulator, using parallelogram mechanism as legs. There are more researches on the dynamic balancing of manipulators which can be found in [15-24].

In this work, theory of design of a 2-DOF planar parallel manipulator is discussed. Three four-bar 2-DOF mechanisms act as the manipulator's legs, and move a thin triangular platform. If all four links of the proposed leg have the same length, then this mechanism can be referred to as pantograph. The organization of the paper is as follows: in the next section, the reactionless conditions of the manipulator is discussed and formulated. Then the theory of design of the manipulator based on these conditions is addressed. Finally, through a numerical example a reactionless manipulator is designed and its dynamic balancing is shown and verified by computer simulation.

## 2 DYNAMIC BALANCING OF THE MECHANISM

The 2-DOF manipulator discussed here is composed of three legs which are connected to the base and to a common thin triangular moving platform Fig. 1. Each leg comprises four links with length of  $L_1, L_2, L_3, L_4$  and masses of  $m_1, m_2, m_3$  and  $m_4$  connected by revolute joints  $j_1, j_2, j_3$  and  $j_4$ . Joint angles  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  are corresponded to links 1, 2, 3 and 4, respectively. It is obviously clear from the Fig. 2 that  $\theta_1 = \theta_2$  and  $\theta_3 = \theta_4$  and  $L_1 = L_3$ . The mass of moving platform is  $M_p$ . To analysis dynamic balancing of this

manipulator, two mentioned conditions are mathematically presented here:

**Table 1** list of symbols

Symbo l	Quantity
M	Shaking force
G	Shaking moment
$r_c$	Velocity of the center of mass of manipulator
m	Total mass of manipulator
$r_a$	Position vector of ith link
$a_i$	Distance of ith counter-weight from its corresponded link
$H_c$	Angular momentum
$H_c^i$	Angular momentum of ith leg about center of mass of the manipulator
$H_{oi}^{ij}$	Angular momentum of jth link of ith leg about its center of mass
$k_i$	Radius of gyration

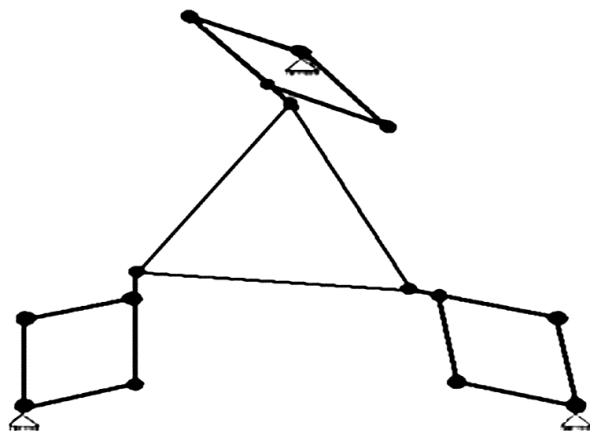
$$\sum F = \partial G / \partial t = G = 0 \tag{1}$$

$$\sum M = \partial_G H_c / \partial t = H_c = 0 \tag{2}$$

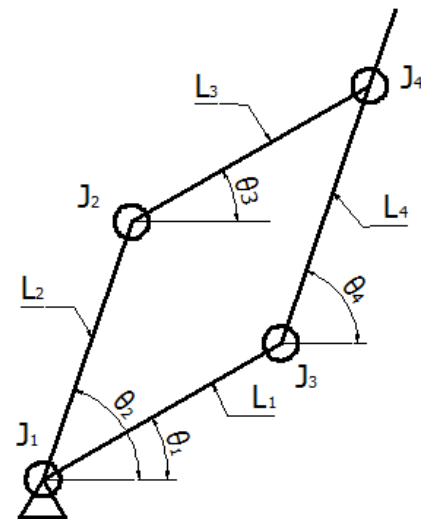
The linear momentum can be considered as Eq. (3):

$$G = mr_c \tag{3}$$

By inertial fixing the center of mass of manipulator and restriction its motion in a plane normal to direction of gravity, the first condition can be satisfied. The procedure of fixing the center of mass of mechanism is discussed in the next section.



**Fig. 1** The 2-DOF planar parallel manipulator with three 2-DOF four-bar legs



**Fig. 2** Four-bar 2-DOF mechanism with revolute joints

### 3 THE FIRST CONDITION

The procedure to satisfy the first condition for the planar parallel manipulator with three legs and triangular platform is as follow:

Distribute the mass of moving platform at three points connected to the legs. This should be done in a way that the effect of inertia and mass of the platform can be replaced by these three points. To satisfy this condition, three conditions which are presented by [22] have to be considered. These conditions are:

The sum of the point mass should be the same as the mass of platform;

The center of mass of platform with respect to a fixed point will be the same as center of mass of the all point masses

The moment of inertia of the point masses about the center of mass of platform is the same as the moment of inertia of platform about its center of mass.

Regarding these conditions, mass of each of these three points would be  $M_p / 3$  From now on, the effect of platform is replaced by these three point masses at end of the legs.

Locate the center of mass of each leg at its base. It can be done by adding three counter-weights, at the opposite extended direction of links 1, 2 and 3 as shown in Fig. 3.

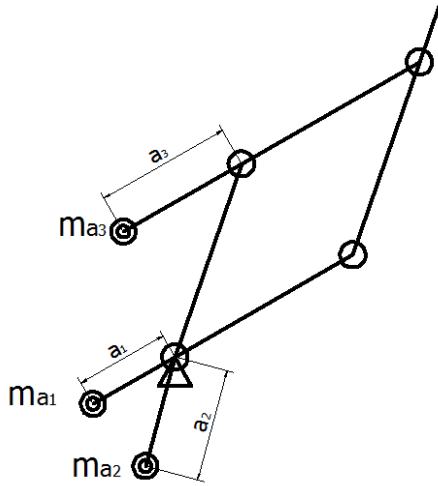


Fig. 3 Locating the center of mass of leg at the base using three counter-weights

The position vector of the center of mass of leg  $r_{oi}$  is:

$$\begin{aligned} Mr_{oi} = & m_1 r_1 e_1 + m_2 r_2 e_2 + m_3 (L_2 e_2 + r_3 e_1) \\ & + m_4 (L_1 e_1 + r_4 e_2) + M_p / 3 (L_1 e_1 + L_4 e_2) - \\ & m_{a1} a_1 e_1 - m_{a2} a_2 e_2 + m_{a3} (L_2 e_2 a_3 e_3) \end{aligned} \quad (4)$$

So these constraints have to be satisfied:

$$m_1 r_1 + m_3 r_3 + m_4 L_1 + \frac{M_p}{3} L_1 - m_{a1} a_1 - m_{a3} a_3 = 0 \quad (5)$$

$$m_2 r_2 + m_3 L_2 + m_4 r_4 + \frac{M_p}{3} L_4 - m_{a2} a_2 - m_{a3} L_2 = 0 \quad (6)$$

#### 4 THE SECOND CONDITION

Time rate of total angular momentum of the manipulator with respect to its center of mass has to be vanished. While the center of mass of manipulator is fixed, total angular momentum of manipulator with respect to its center of mass has to be fixed i.e.:

$$H_c = \text{Constant} = 0 \quad (7)$$

With this in mind that effect of platform is replaced by three point masses at end of each leg, so total angular momentum would be:

$$H_c = \sum_{i=1}^3 H_c^i \quad (8)$$

The angular momentum of  $i$ th link with respect to center of mass of manipulator is related to  $i$ th link about its base:

$$H_c^i = H_{oi}^i + r_{coi} \times m_i \dot{r}_{coi} \quad (9)$$

The center of mass of manipulator is fixed as well as base of the legs. Thus the velocity vector  $\dot{r}_{coi}$  vanishes:

$$H_c^i = H_{oi}^i \quad (10)$$

Angular momentum of  $i$ th leg about its base can be rewritten as:

$$H_{oi}^i = \sum_{j=1}^4 H_{oi}^{ij} + r_p \times \frac{M_p}{3} \dot{r}_p + \sum_{i=1}^3 a_i \times m_{ai} \dot{a}_i \quad (11)$$

Where:

$$a_1 = -a_1 e_1 \quad (12)$$

$$a_2 = -a_2 e_2 \quad (13)$$

$$a_3 = L_2 e_2 - a_3 e_3 \quad (14)$$

And

$$r_p = L_1 e_1 - L_4 e_2 \quad (15)$$

The total angular momentum is a vector which is parallel to the gravity vector. Regarding to Cartesian coordinate, the angular momentum of  $i$ th leg can be reevaluated as:

$$H_{oi}^i = A \theta_1 + B \theta_2 + C \cos(\theta_1 - \theta_2) (\theta_1 + \theta_2) \quad (16)$$

$$\begin{aligned} A = & (m_{i1} K_1^2 + m_{i1} r_1^2 + m_{i3} K_3^2 + m_{i3} L_1^2 \\ & + m_{i4} r_4^2 + m_{a1} a_1^2 + m_{a3} a_3^2 + \frac{M_p}{3} L_4^2) \end{aligned} \quad (17)$$

$$\begin{aligned} B = & (m_{i2} K_2^2 + m_{i2} r_2^2 + m_{i3} r_3^2 + m_{i4} K_4^2 + m_{i4} L_2^2 \\ & + m_{a2} a_2^2 + m_{a3} L_3^2 + \frac{M_p}{3} L_1^2) \end{aligned} \quad (18)$$

$$C = (m_{i3} L_1 r_3 + m_{i4} L_2 r_4 - m_{a3} L_2 a_3 + \frac{M_p}{3} L_1 L_4) \quad (19)$$

The total angular momentum can be vanished by proper choice of geometrical parameters and adding counter-rotations or through a proper trajectory planning. While the total angular momentum of the system about its center of mass is the same as sum of angular momentums of legs about their bases, thus, in the first method, angular momentum of each leg about its base has to be vanished.

$$H_{oi}^i = 0; i = 1, 2, 3 \quad (20)$$

To vanish angular momentum of each leg, two counter-rotations are mounted at the base of each leg. The inertia parameter of each counter-rotation is as follows:

$$l_1 = (m_{i1}K_1^2 + m_{i1}r_1^2 + m_{i3}K_3^2 + m_{i3}L_1^2 + m_{i4}r_4^2 + m_{a1}a_1^2 + m_{a3}a_3^2 + \frac{M_p}{3}L_4^2) \quad (21)$$

$$l_2 = (m_{i2}K_2^2 + m_{i2}r_2^2 + m_{i3}r_3^2 + m_{i4}K_4^2 + m_{i4}L_2^2 + m_{a2}a_2^2 + m_{a3}L_3^2 + \frac{M_p}{3}L_1^2) \quad (22)$$

Also this constraint must be considered:

$$m_{i3}L_1r_3 + m_{i4}L_2r_4 - m_{a3}L_2a_3 + \frac{M_p}{3}L_1L_4 = 0 \quad (23)$$

### 5 NUMERICAL EXAMPLE

In this section, a numerical example is proposed. The required data is presented in Table 2. The parameters which are desired to be obtained would be mass of counter-weights, distance of each counter-weight from its connected link as well as inertial parameters of counter-rotations. However, there will be several answers sets to this example, by defining a proper objective function and applying an optimization procedure, best answer set could be found. Interested readers may refer to [5].

**Table 2** The data set of proposed example

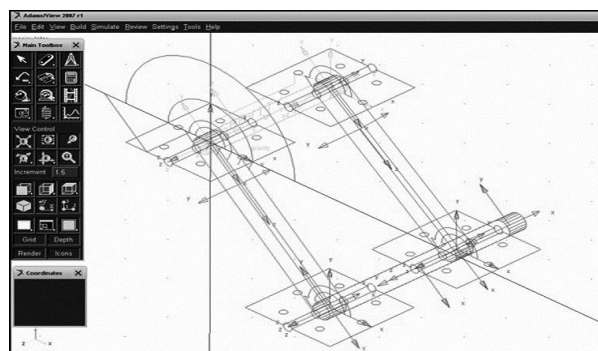
Member	Length/Area	Mass	Radius of gyration
Link1	0.2 m	0.18 Kg	0.058 m
Link2	0.2 m	0.18 Kg	0.058 m
Link3	0.2 m	0.18 Kg	0.058 m
Link4	0.3 m	0.27 Kg	0.087 m
Platform	0.0017 m <sup>2</sup>	0.3 Kg	-

Regarding the obtained equations, the answer set is as follow: (However, other answer sets can be considered, too.)

$$\{ a_1=0.2 \text{ (m)}; m_{a1}=0.1075 \text{ (Kg)}; a_2=0.2 \text{ (m)}; m_{a2}=1.065 \text{ (Kg)}; a_3=0.2 \text{ (m)}; m_{a3}=0.4425 \text{ (Kg)}; l_1=0.04729 \text{ (Kg}m^2\text{)}; l_2=0.08134915 \text{ (Kg}m^2\text{)} \}$$

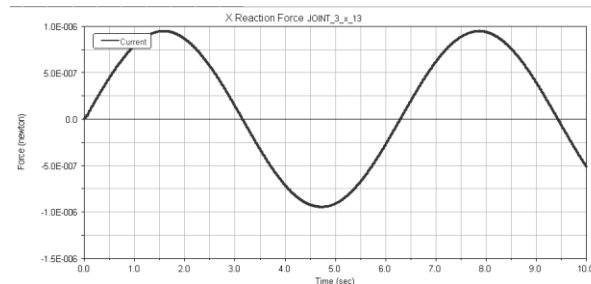
The verification of dynamic balancing of mechanism is performed using ADAMS software. By defining the mechanism in ADAMS/View environment, as shown in Fig 4. and then applying obtained answer set to the mechanism, dynamic simulation is performed for several trajectories. It should be noted that counter-weights and counter-rotations are not shown in this view. To verify dynamic balancing property, the reaction forces applied to the base are plotted during

simulated ten seconds and reaction moments applied to the base are plotted during simulated ten seconds shown in Fig. 5 & Fig. 6 & Fig. 7.

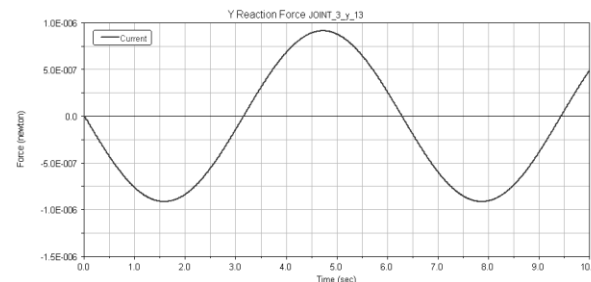


**Fig. 4** Simulating 2-DOF four-bar mechanism acting as manipulator's leg in ADAMS

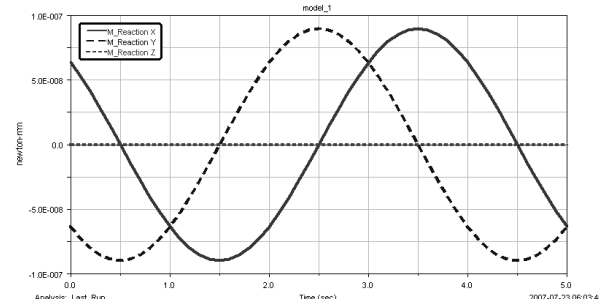
As one could see, it is obviously clear from blow figures, that reaction forces and moments are very small.



**Fig. 5** X Reaction force applied to base during ten seconds



**Fig. 6** Y Reaction force applied to base during ten seconds



**Fig. 7** Reaction moments applied to base during five seconds

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## 6 CONCLUSION

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This paper provided the design and theory for a dynamically balanced planar parallel mechanism. The system center of mass was inertial fixed using counterweights. Therefore, the total linear momentum and angular momentum of the system vanished using proper choice of inertia and geometric parameters of counter-rotations mounted at the base of each leg. Designed mechanism was tested using ADAMS software and it was shown that the proposed manipulator is dynamically balanced.

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